Statistics

Spring 2023

Lecture 8

Feb 19-8:47 AM

Multiplication Rule:

1) Independent events

$P(A \text{ and } B) = P(A) \cdot P(B)$

ex: $P(A) = .4$, $P(B) = .7$, A&B are independent events

$P(\bar{A}) = 1 - P(A) = .6$

$P(\bar{B}) = 1 - P(B) = .3$

$P(A \text{ and } B) = P(A) \cdot P(B) = (.4)(.7) = \boxed{.28}$

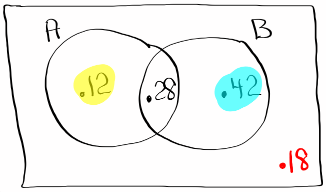
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .4 + .7 - .28 = \boxed{.82}$

Venn Diagram

$.4 - .28 = .12$

$.7 - .28 = .42$



$P(A \text{ only} \text{ OR } B\text{only}) = .12 + .42 = \boxed{.54}$ Total = 1

Mar 28-6:50 PM

A full-deck of playing cards has 52 Cards and 12 face Cards.

Draw 2 Cards with replacement.

$F F \qquad F \bar{F} \qquad \bar{F} F \qquad \bar{F} \bar{F}$

$P(\text{2 face Cards}) = \frac{12}{52} \cdot \frac{12}{52} = \frac{3}{13} \cdot \frac{3}{13} = \boxed{\frac{9}{169}}$

$P(\text{exactly 1 face Card}) = 2 \cdot \frac{12}{52} \cdot \frac{40}{52} = \boxed{\frac{60}{169}}$

$P(\text{No face Cards}) = \frac{40}{52} \cdot \frac{40}{52} = \boxed{\frac{100}{169}}$

	# Face	P(#Face)	
	2	9/169	
L1	1	60/169	L2
	0	100/169	

clear all lists

#Face → L1

P(#Face) → L2

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2 to find

$\bar{x} = .462$

S = Blank

n = 1 ← Total Prob. = 1

Mar 28-6:57 PM

There are 5 Females and 7 males.

we need to select 3 people

Order does not matter.

1) How many ways can this be done?

$_{12}C_3 = 220$

2) How many ways can be select 3 females?

$_5C_3 \cdot {}_7C_0 = 10$

3) P(All Selections are females) =

$P(FFF) = \frac{_5C_3 \cdot {}_7C_0}{_{12}C_3} = \frac{10}{220} = \boxed{\frac{1}{22}}$

Sample Space

FFF

FFM

FMF

FMM

MFF

MFM

MMF

MMM

Mar 28-7:07 PM

$P(2 \text{ Females } \& \ 1M) = \frac{{}_5C_2 \cdot {}_7C_1}{{}_{12}C_3} = \frac{70}{220} = \boxed{\frac{7}{22}}$

$P(1 \text{ Female } \& \ 2 \text{ Males}) = \frac{{}_5C_1 \cdot {}_7C_2}{{}_{12}C_3} = \frac{105}{220} = \boxed{\frac{21}{44}}$

$P(\text{No females}) = P(\text{All males}) = \frac{{}_5C_0 \cdot {}_7C_3}{{}_{12}C_3} = \frac{35}{220} = \boxed{\frac{7}{44}}$

# Females	P(# Females)
3	1/22
2	7/22
1	21/44
0	7/44

L1 L2

clear all lists

Females → L1

P(# Females) → L2

use $\boxed{\text{1-Var Stats}}$ with

L1 & L2 to find

$\bar{x} = 1.25$

S = blank

n = 1

Mar 28-7:13 PM

$P(\text{at least 1 Female}) = 1 - P(\text{No Females})$

↑

Total Prob.

$= 1 - P(\text{All Males})$

$= 1 - \frac{7}{44} = \boxed{\frac{37}{44}}$

$P(\text{at least 1 Male}) = 1 - P(\text{No male})$

$= 1 - P(\text{All Females})$

$= 1 - \frac{1}{22} = \boxed{\frac{21}{22}}$

Mar 28-7:22 PM

Suppose There are 5 dimes & 10 nickels.
Select 2 Coins, No replacement.

DD → 20¢

$\underbrace{DN \qquad ND}$ 15¢

NN ↑ 10¢

$P(20¢) = \frac{5}{15} \cdot \frac{4}{14} = \boxed{\frac{2}{21}}$

$P(15¢) = 2 \cdot \frac{5}{15} \cdot \frac{10}{14} = \boxed{\frac{10}{21}}$

$P(10¢) = \frac{10}{15} \cdot \frac{9}{14} = \boxed{\frac{3}{7}}$

Total ¢	P(Total ¢)
20¢	2/21
15¢	10/21
10¢	3/7

L1 L2

clear all lists
Total ¢ → L1
P(Total ¢) → L2
use $\boxed{\text{1-Var Stats}}$
with L1 & L2 to find

$\bar{x} = 13.\overline{3}$ S= blank $n = 1$

Mar 28-7:26 PM

Multiplication Rule:

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

A happens,
then B happens

Given

ex: Draw 2 Cards from a full deck of Playing cards without replacement.

$P(\text{both are Aces}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

$P(\text{both are face Cards}) = \frac{12}{52} \cdot \frac{11}{51} = \frac{11}{221}$

Suppose we draw 3 Cards,

$P(\text{All Red Cards}) = \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} = \boxed{\frac{2}{17}}$

Mar 28-7:36 PM

A box has 3 red, 7 blue, and 10 white balls.

Take 3 balls, no replacement.

$$P(\text{Red, Blue, then White}) = \frac{3}{20} \cdot \frac{7}{19} \cdot \frac{10}{18} = \boxed{\frac{7}{228}}$$

$$P(\text{3 Red Balls}) = \frac{3}{20} \cdot \frac{2}{19} \cdot \frac{1}{18} = \boxed{\frac{1}{1140}}$$

$$P(\text{at least 1 Red}) = 1 - P(\text{No Reds})$$

RRR

Some R

Some $\bar{R}$

$\bar{R}\,\bar{R}\,\bar{R}$

$$= 1 - \frac{17}{20} \cdot \frac{16}{19} \cdot \frac{15}{18}$$

$$= \boxed{\frac{23}{57}} \checkmark$$

$$P(\text{at least 1 Blue}) = 1 - P(\text{No Blue})$$

B B B

Some B

&

Some $\bar{B}$

$\bar{B}\,\bar{B}\,\bar{B}$

$$= 1 - \frac{13}{20} \cdot \frac{12}{19} \cdot \frac{11}{18}$$

$$= \boxed{\frac{427}{570}}$$

Mar 28-7:42 PM

From multiplication rule

$$P(A \text{ and } B) = P(A) \cdot \boxed{P(B|A)}$$

If we solve for P(B|A), we get

$$\boxed{P(B|A) = \frac{P(A \text{ and } B)}{P(A)}}$$

Conditional Prob.

ex: $P(A) = .8$

$P(B) = .7$

$P(A \text{ and } B) = .6$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{.6}{.8} = \boxed{.75}$$

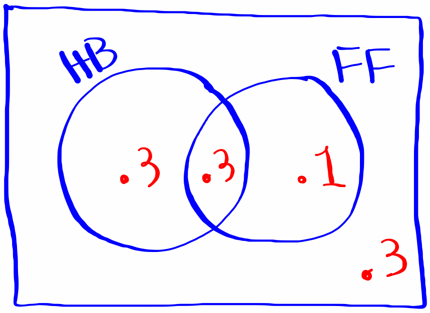
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{.6}{.7} = \boxed{.857}$$

Mar 28-8:03 PM

$P(HB) = .6$

$P(FF) = .4$

$P(HB \text{ and } FF) = .3$



Total = 1

$$P(FF \mid HB) = \frac{P(HB \text{ and } FF)}{P(HB)} = \frac{.3}{.6} = \boxed{.5}$$

$$P(HB \mid FF) = \frac{P(HB \text{ and } FF)}{P(FF)} = \frac{.3}{.4} = \boxed{.75}$$

Mar 28-8:08 PM

$P(\text{Shoes}) = .5$

$P(\text{Pants}) = .6$

$P(\text{Shoes} \mid \text{Pants}) = .8$

$P(\text{Shoes and Pants})$

$$P(\text{Shoes} \mid \text{Pants}) = \frac{P(\text{Shoes \& Pants})}{P(\text{Pants})}$$

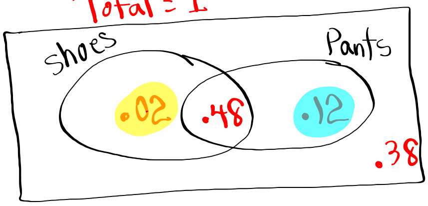
$$.8 = \frac{P(\text{Shoes and pants})}{.6}$$

$$P(\text{Shoes and pants}) = \boxed{.48}$$

$$P(\text{Pants} \mid \text{Shoes}) = \frac{P(\text{Shoes and Pants})}{P(\text{Shoes})}$$

$$= \frac{.48}{.5} = \boxed{.96}$$

Total = 1



$P(\text{Shoes only or Pants only})$

$= .02 + .12 = \boxed{.14}$

Mar 28-8:12 PM

P(math) = .5

P(English) = .75

P(math | English) = .8

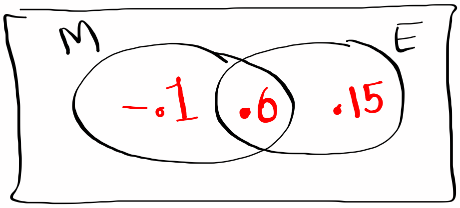
P(Math and English)

$$P(M|E) = \frac{P(M \text{ and } E)}{P(E)}$$

$$.8 = \frac{P(M \text{ and } E)}{.75}$$

Cross-multiply

P(M and E) = .6



Impossible

Mar 28-8:19 PM

P(Math) = .4

P(English) = .5

P(Math | English) = .6

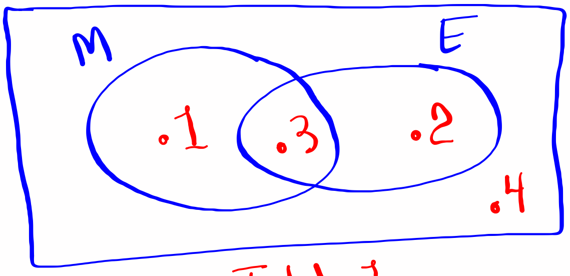
P(Math and English)

$$P(M|E) = \frac{P(M \text{ and } E)}{P(E)}$$

$$.6 = \frac{P(M \text{ and } E)}{.5}$$

$$\boxed{P(M \text{ and } E) = .3}$$

SG 13 ✓



Total = 1

Mar 28-8:24 PM

Complete the Chart below:

x	$P(x)$	$x\,P(x)$	$x^2 P(x)$
1	.2	.2	.2
2	.5	1.0	2.0
3	.3	.9	2.7

Find

1) $\sum P(x) = 1$

2) $\sum x p(x) = 2.1$

3) $\sum x^2 p(x) = 4.9$

4) Compute

$$\sum x^2 p(x) - \left[\sum x p(x)\right]^2 =$$

$$4.9 - 2.1^2 = \boxed{.49}$$

5) $\sqrt{\text{last answer}} = \sqrt{.49} = \boxed{.7}$

Mar 28-8:28 PM

Complete the Chart below:

x	$P(x)$	$x\,P(x)$	$x^2 p(x)$
1	.1	.1	.1
2	.2	.4	.8
3	.3	.9	2.7
4	.4	1.6	6.4

1) $\sum P(x) = 1$

2) $\sum x p(x) = 3$

3) $\sum x^2 p(x) = 10$

4) Compute $\sum x^2 p(x) - \left[\sum x p(x)\right]^2$

$$= 10 - 3^2 = \boxed{1}$$

5) Find $\sqrt{\text{last answer}} = \sqrt{1} = \boxed{1}$

Mar 28-8:33 PM

Data
- 1) Qualitative
- 2) Quantitative
  - 1) Discrete
    countable
  - 2) Continuous
    Measureable

Mar 28-8:39 PM

Let x be a discrete random variable with Prob. dist. P(x),

what is prob. dist.?

It is a way to give prob. of all possible outcomes.

It could be in form of

1) a table

2) formula

3) graph

4) Basic concepts of prob.

Mar 28-8:41 PM

Terms & Conditions for Prob. dist. $P(x)$:

1) $0 \le P(x) \le 1$

2) Sum of all prob. is always 1.

3) $P(x) = 1 \iff$ Sure event

4) $P(x) = 0 \iff$ Impossible event

5) $0 < P(x) \le .05 \iff$ Rare event

Mar 28-8:44 PM

Mean μ "mu"

Variance σ^2 "sigma^2"

Standard deviation σ "sigma"

$$\boxed{\begin{aligned} \mu &= \sum x\,p(x) \\ \sigma^2 &= \sum x^2 p(x) - \mu^2 \\ \sigma &= \sqrt{\sigma^2} \end{aligned}}$$

Mar 28-8:46 PM

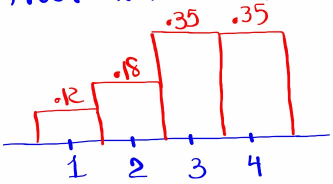
Consider the chart below:

x	$P(x)$
1	.12
2	.18
3	.35
4	.35

1) verify $\sum P(x) = 1$

$.12 + .18 + .35 + .35 = 1$ ✓

2) Prob. dist. Histogram



$x \to L1$, $P(x) \to L2$

Use [1-Var Stats]

with L1 & L2

$\mu = \bar{x} = 2.93$

$\sigma = \sigma_x = 1.003$

$n = 1$ ✓

For σ^2

[VARS] [5: Statistics]

[4: σ_x] [x^2] [MATH]

[1:▶Frac] [Enter]

$\sigma^2 = 1.0051$

$$\boxed{\frac{10051}{10000}}$$

Mar 28-8:49 PM

Consider the chart below:

x	$P(x)$
1	.05
2	.15
3	.3
4	.4
5	.1

1) Find $P(x=5)$

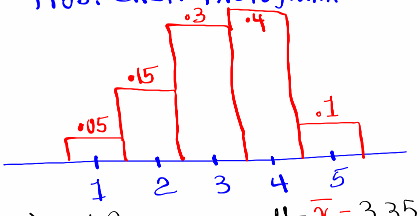
$= 1 - [.05 + .15 + .3 + .4]$

$= \boxed{.1}$

2) $P(x=2 \text{ or } x=4) =$

$.15 + .4 = \boxed{.55}$

3) Draw Prob. dist. Histogram.



$x \to L1$, $P(x) \to L2$

Use [1-Var Stats]

with L1 & L2 to find

$\mu = \bar{x} = 3.35$

$\sigma = \sigma_x = 1.014$

$n = 1$ ✓

$\boxed{\sigma^2 = \frac{411}{400}}$

Find σ^2 in reduced fraction

[VARS] [5: Statistics] [4: σ_x] [x^2] [Math] [1:▶Frac]

[Enter]

Mar 28-8:56 PM

Application:

Expected Value $\rightarrow \mu \rightarrow \bar{x}$

40 TKTS were Sold for \$10 each.

One ticket is drawn, owner of this ticket gets a Calc worth \$100.

Expected Value per ticket.

Net	P(Net)	
10 − 100	1/40	winning
10 − 0	39/40	losing

Net → L1

P(Net) → L2

1-Var Stats with L1 & L2

Expected Value $= \mu = \bar{x}$

\$7.50/TKT

Mar 28-9:05 PM

You are taking a flight.

You pay \$100 for insurance to luggages.

Any damages, Airline pays You \$1000

P(damage) = .5%   Find expected Value Per Policy Sold.

Net	P(Net)	
100 − 1000	.5% = .005	Damage
100 − 0	.995	No Damages

Net → L1

P(Net) → L2

1-Var Stats with L1 & L2

$E.V. = \mu = \bar{x} = 95$

\$95/Policy

SG 14 & SG 15

Mar 28-9:11 PM